Transient Solutions of Fokker-Planck Equation in a Paul Trap

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Abstract

In this work, we have analyzed the Fokker-Planck equation for Plasma in a Paul Trap in more detail and found two interesting features of the solutions regarding number of solutions and their behaviour.

Firstly, for a time-periodic spatially linear electric field, the Fokker-Planck equation seems to admit solutions only of the form of a Gaussian (quadratic polynomial in the spatial and velocity coordinates raised to an exponential). If we take higher powers of the spatial and velocity coordinates in the exponential, their coefficients turn out to be zero even for the transient state.

Secondly when we assume a similar Gaussian solution for the Fokker-Planck equation, we find that the coefficients of the three terms inside the exponential have two fixed points, one of which is the well known stable Boltzmann solution. However, there is another fixed point, which is unstable and there could be special initial conditions in the plasma distribution, for which these coefficients blow up with time.

Introduction

Paul trap is a device used to confine charged particles of the same species by using **time-periodic spatially linear electric fields.**

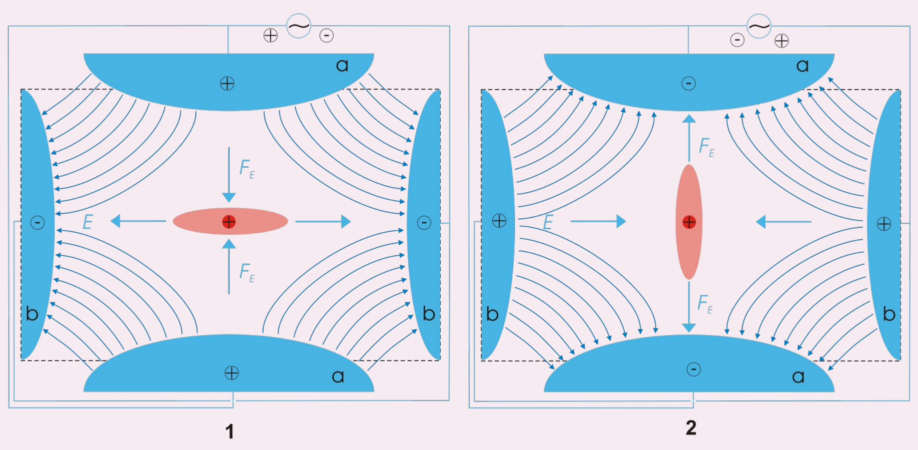


Fig.1[1] Fields inside a Paul Trap

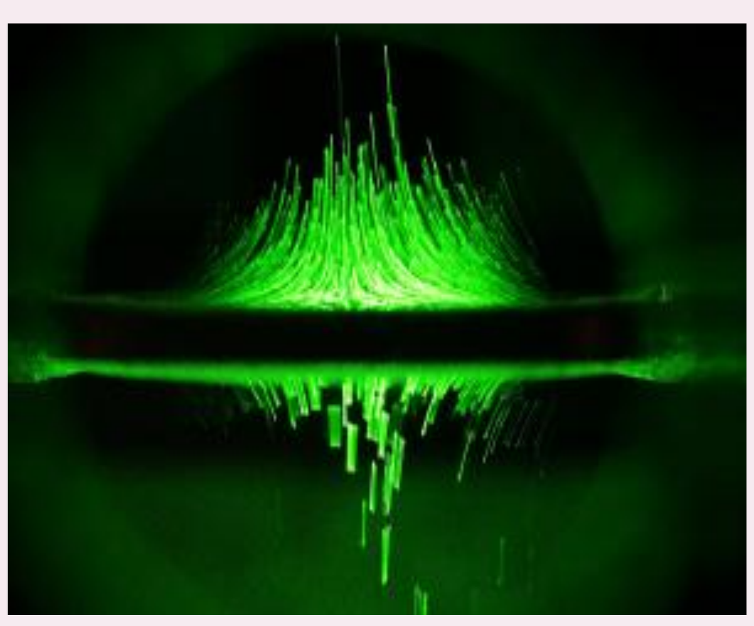


Fig.2[1] Charged Particles inside Paul Trap

The collision-less **Vlasov equation** for spatially non-uniform time-periodic electric fields is known to have **infinitely many solutions**, most of which are **quasi-periodic** in time[1-2].

However, it is usually believed that the solutions of the collisional Fokker-Planck equation for such systems asymptotically reach an equilibrium state of time-periodicity irrespective of the initial conditions [3].

Description

- Working with Fokker Planck Equation for Plasma in a Paul Trap by introducing a Noise/Diffusion term and a Friction Term in to it.[1]
- We wanted to see if this kind of equation can have multiple Stable & Normalizable Solutions and do they all behave Asymptotically same for different initial conditions in the distribution function and where do they reach after a long time of evolution.
- The equation is as Follows:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + R^2\frac{\partial f}{\partial v} = \gamma\frac{\partial (vf)}{\partial v} + \delta\frac{\partial^2 f}{\partial v^2}$$

- Considered following R^2 for the above equation:

  1.(−p + 2qCos(2t))x
  2.(−qxCos(2t))

- Considered the following Solution for the differential equation.

$$f = g(t)e^{-\sum_{i=0}^{6} A_i(t)v^{6-i}x^i - \sum_{i=0}^{6} B_i(t)v^{4-i}x^i - \sum_{i=0}^{2} C_i(t)v^{2-i}x^i}$$

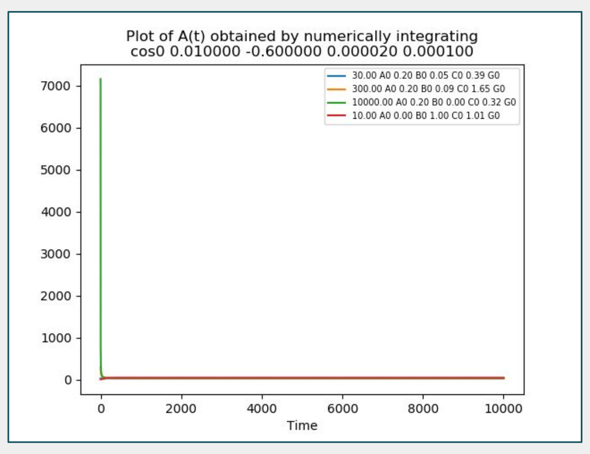
Results

1. We found only a single set of values where the distribution function arrives for certain set of initial conditions, they become unstable when stated from other than the stable initial conditions.
2. They were supposed to arrive at the other set of solutions at those initial conditions (other then the one from above set.), but this happens only when the coefficient of 'vx' is not evolved with time.

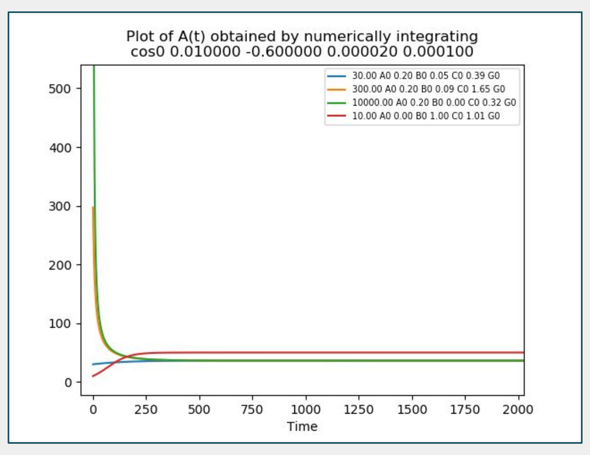
B	0	q/δ
A	$\gamma/(2\delta)$	$(\gamma \pm (\gamma^2 - 4q)^{1/2}) / (4\delta)$
C	$\gamma q/(2\delta)$	$(-q(\gamma \pm (\gamma^2 - 4q)^{1/2}) / (4\delta)) + (\gamma q/(2\delta))$

Graphs

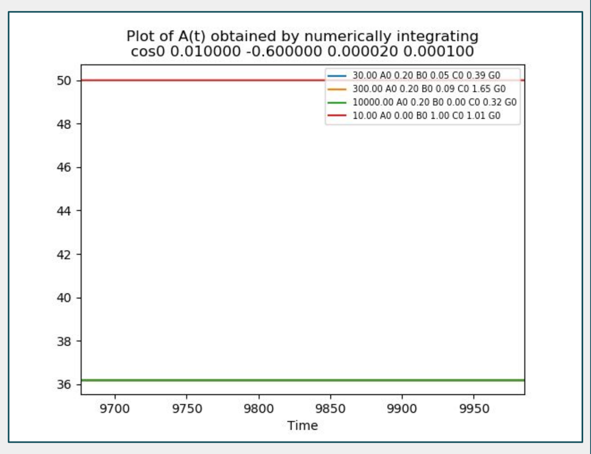
i. This are the graph of A(t).



Original



Zoomed



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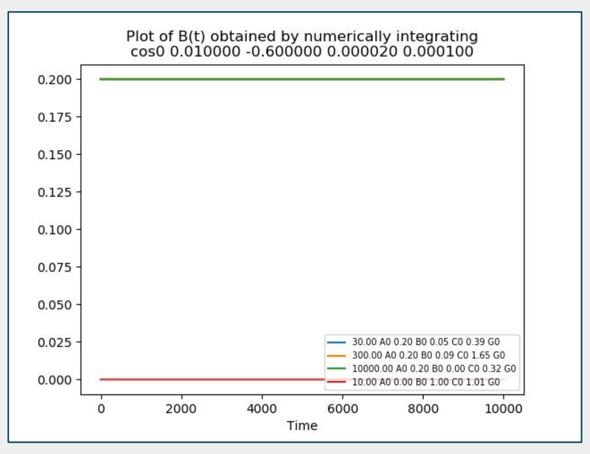
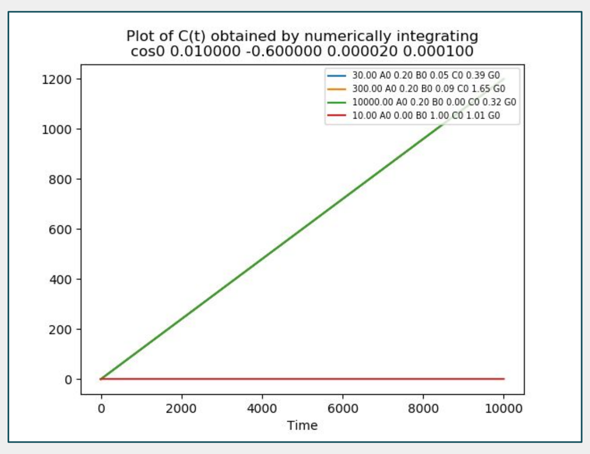
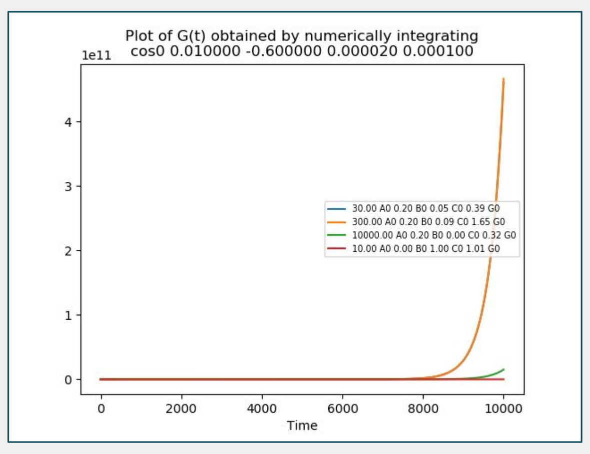
ii. This is the graph of B(t).



iii. This is the graph of C(t).



iv. This is the graph of G(t).



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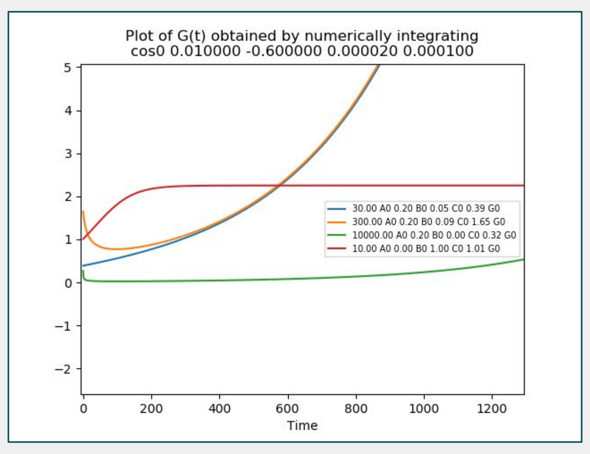

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Conclusions

- There is something happening when coefficient of '**vx**' is kept constant, could lead us to other solutions too but at the cost of **g(t) getting blown up**.
- In later work[4] it was shown that there are such **regions of stable and unstable solutions** where the distribution function starts from **unstable initial solutions** and then **returns back to the stable zone** after sometime.
- The primary conclusion of our work is that the transient solutions of Fokker-Planck equation need to be carefully analyzed and merely looking at the equilibrium solutions can be misleading.

References

1. Analytic, nonlinearly exact solutions of an rf confined plasma, *Phys. Plasmas* **15**, 062303 (2008) https://doi.org/10.1063/1.2926632
2. Vlasov dynamics of periodically driven systems, *Phys. Plasmas* **25**, 042302 (2018)
3. Asymptotic solution of Fokker-Planck equation for plasma in Paul traps, *Phys. of Plasmas* **17**,054501 https://doi.org/10.1063/1.3418373
4. arXiv:1809.10952 [physics.plasm-ph]

Image Credits

1. https://en.wikipedia.org/wiki/Quadrupole_ion_trap
2. https://aquadrupauliontrap.wordpress.com